INVESTIGATION OF THE MOTION OF A SPACE VEHICLE BY THE METHODS OF SIMILARITY THEORY

PMM Vol. 35, №5, 1971, pp. 930-932
E. E. IVANOVA
(Moscow)
(Received January 1, 1971)

For the generalization of numerical results in the investigation of the motion of space vehicles in a central gravitational field, a change to the dimensionless form of the differential equations is effected by the introduction of similarity coefficients for the basic parameters of the motion. With the use of similarity coefficients the solution of the problem is readily evaluated for arbitrary geometric dimensions of the orbit and characteristics of the gravity field for fixed values of the eccentricity The introduction of similarity coefficients is valid for the consideration of either relative or absolute motion of a space vehicle.

The relative motion of an active interceptor space vehicle in its approach to a passive target vehicle is described by a system of nonlinear differential equations with variable coefficients.

Analytic investigation of this system is impossible in the general case, so that it is necessary to use numerical methods. The solution depends on the shape of the orbit of the target (the eccentricity e), its linear dimensions (the focal parameter p), the characteristics of the gravitational field (the gravitational constant μ) and the initial conditions (the relative coordinates and velocity of the interceptor at the initial instant of the maneuver).

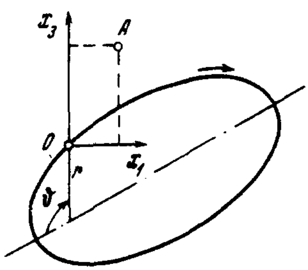

Fig. 1.

The longitudinal relative motion of the interceptor A in a system of coordinates connected with the target O, for a central gravitational field (Fig. 1) is expressed by the system [1]

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = \frac{\mu p}{r^4} x_1 + 2\frac{\mu e}{r^3}(\sin\vartheta)\, x_3 - 2\frac{\sqrt{\mu p}}{r^2} x_4 - \mu\frac{x_1}{[x_1^2 + (r + x_3)^2]^{3/2}}$$

$$\frac{dx_3}{dt} = x_4 \qquad \left(r = \frac{p}{1 + e\cos\vartheta}\right) \tag{1}$$

$$\frac{dx_4}{dt} = \frac{\mu}{r^2} - 2\frac{\mu e}{r^3}(\sin\vartheta)\, x_1 + 2\frac{\sqrt{\mu p}}{r^2} x_2 + \frac{\mu p}{r^4} x_3 - \mu\frac{r + x_3}{[x_1^2 + (r + x_3)^2]^{3/2}}$$

Here r is the distance from the target to the gravitational center, x_1 and x_3 are the relative coordinates of the interceptor, x_2 and x_4 are the components of the relative velocity of the interceptor, t is the time, and ϑ is the true anomaly of the target reckoned from the pericenter. The sectoral speed of the target O is constant, so that the true anomaly changes rapidly near the pericenter and slowly in the vicinity of the apocenter. As a result the variable coefficients of system (1), depending on the time t, change nonuniformly. If the true anomaly ϑ is taken as the argument in the system (1), the variable coefficients of the equations will change more uniformly, which for a given accuracy of the solution permits the integration steps to be increased and the machine time to be reduced.

The change to the new argument can be carried out with the aid of the relation

$$dt/d\vartheta = K_t/\lambda^2 \qquad (K_t = \sqrt{p^3/\mu}, \quad \lambda = 1 + e\cos\vartheta) \tag{2}$$

Then the system (1) takes the form

$$\begin{gathered}
\frac{dx_1}{d\vartheta} = \frac{K_t}{\lambda^2} x_2 \\
\frac{dx_2}{d\vartheta} = \frac{1}{K_t}\left[\lambda^2 x_1 + 2e\lambda(\sin\vartheta)\, x_3 - \frac{p^3}{\lambda^2}\frac{x_1}{[x_1^2 + (p/\lambda + x_3)^2]^{3/2}}\right] - 2x_4 \\
\frac{dx_3}{d\vartheta} = \frac{K_t}{\lambda^2} x_4 \\
\frac{dx_4}{d\vartheta} = \frac{1}{K_t}\left[-2e\lambda(\sin\vartheta)\, x_1 + \lambda^2 x_3 - \frac{p^3}{\lambda^2}\frac{p/\lambda + x_3}{[x_1^2 + (p/\lambda + x_3)^2]^{3/2}}\right] + 2x_2 + \sqrt{\mu/p}
\end{gathered} \tag{3}$$

In a series of works, for example [2, 3], transformation to dimensionless quantities is proposed for the simplification of the equations of relative motion. However, the use as unit of length of the distance from the target 0 to the gravitational center [2], which changes in the course of the motion, or of the mean anomaly as the argument [3], does not permit generalization of the solution of a problem of the approach os space vehicles.

If the units of length and speed are taken as

$$K_l = p, \quad K_v = \sqrt{\mu/p} \tag{4}$$

respectively, then the system (3) in dimensionless form becomes

$$\begin{gathered}
\frac{dX_1}{d\vartheta} = \frac{1}{\lambda^2} X_2 \quad \left(X_1 = \frac{x_1}{K_l}, \quad X_2 = \frac{x_2}{K_v}\right) \\
\frac{dX_2}{d\vartheta} = \lambda^2 X_1 + 2e\lambda(\sin\vartheta)\, X_3 - \frac{X_1}{\lambda^2[X_1^2 + (1/\lambda + X_3)^2]^{3/2}} - 2X_4 \\
\frac{dX_3}{d\vartheta} = \frac{1}{\lambda^2} X_4 \quad \left(X_3 = \frac{x_3}{K_l}, \quad X_4 = \frac{x_4}{K_v}\right) \\
\frac{dX_4}{d\vartheta} = 1 - 2e\lambda(\sin\vartheta)\, X_1 + \lambda^2 X_3 - \frac{1/\lambda + X_3}{\lambda^2[X_1^2 + (1/\lambda + X_3)^2]^{3/2}} + 2X_2
\end{gathered} \tag{5}$$

Here X_1 and X_3 are the dimensionless relative coordinates of the interceptor, and X_2 and X_4 are the components of its dimensionless relative velocity.

The system (5) is similar to the system (3), where the similarity coefficients [4] for the relative coordinates and velocity are determined by the expressions (4), and the values of the argument ϑ are equal at corresponding points of similar orbits (with an identical eccentricity e). The determining parameters for the system (3) are μ, p, e, ϑ, x_{10}, x_{20}, x_{30}, x_{40}, and for the system (5) they are e, ϑ, X_{10}, X_{20}, X_{30}, X_{40} (where the suoscript 0 — "zero" — indicates the initial conditions).

The right-hand sides of the system of equations (5) contain only one parameter, the eccentricity e that determines the form of the orbit of the target. If a numerical solution of the problem is known in dimensional form, it can be reinterpreted for the orbit of a target with the same eccentricity but different geometrical dimensions and characteristics of the gravitational field through the ratios of the similarity coefficients:

$$x_1'' = \frac{K_l''}{K_l'} x_1', \quad x_2'' = \frac{K_v''}{K_v'} x_2', \quad x_3'' = \frac{K_l''}{K_l'} x_3', \quad x_4'' = \frac{K_v''}{K_v'} x_4' \tag{6}$$

It can be shown that the similarity coefficient for acceleration is

$$K_w = \mu / p^2 \tag{7}$$

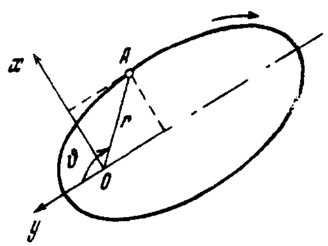

Fig. 2.

The transformation in given solutions of the systems (3) and (5) from the true anomaly to time is effected by known methods [5, 6]. The similarity coefficient K_t for time, the value of which was given early in the expression (2), can be obtained, for example, from the ratio of the periods of rotation of the target through its orbit in the dimensional and dimensionless forms.

The similarity coefficients obtained for relative motion of a space vehicle in the approach problem prove to be applicable also to the consideration of motion in an absolute (planetocentric) system of coordinates (O being the gravitational center and A the space vehicle). In fact, the absolute coordinates (Fig. 2)

$$x = p\frac{\cos\vartheta}{1+e\cos\vartheta}, \qquad y = p\frac{\sin\vartheta}{1+e\cos\vartheta} \tag{8}$$

and the components of the velocity of the vehicle

$$v_x = \sqrt{\mu/p}\,(1+e\cos\vartheta), \qquad v_y = \sqrt{\mu/p}\,\sin\vartheta \tag{9}$$

can, with the use of the similarity coefficients (4), be expressed in the dimensionless form

$$X = \frac{\cos\vartheta}{1+e\cos\vartheta}, \qquad Y = \frac{\sin\vartheta}{1+e\cos\vartheta} \tag{10}$$

$$V_x = 1+e\cos\vartheta, \qquad V_y = e\sin\vartheta \tag{11}$$

The indicated similarity coefficients permit dimensionless equations of motion to be obtained for a space vehicle in an absolute coordinate system. The solution of these equations for an orbit with fixed eccentricity is general in the sense that with the use of the similarity coefficients it is extended to orbits with arbitrary geometric dimensions and values of the gravitational constants.

BIBLIOGRAPHY

1. Ponomarev, V. M., Theory of the Control of Space Vehicles. "Nauka", Moscow, 1965.
2. Ermilov, Iu. A., On the calculation of impulses for the approach of satellites in a central Newtonian gravitational field. Kosmicheskie issledovaniia, Vol. 6, №6, 1968.
3. Antoni, Sazaki., The problem of rendezvous in nearly circular orbits. Raketn. tekhn. i kosmonavtika, Vol. 3, №9, 1965.
4. Sedov, L. I., Methods of Similitude and Dimensions in Mechanics. "Nauka", Moscow, 1967.

5. Duboshin, G.N., Celestial Mechanics. Basic Problems and Methods. "Nauka" Moscow, 1968.
6. Erike, K., Space Flight. Vol. 1, Fizmatgiz, Moscow, 1963.

Translated by M.D.V.D.

ON DISSIPATIVE BEHAVIOR OF AN EQUATION OF NONLINEAR OSCILLATIONS

PMM Vol. 35, №5, 1971, pp. 933-937
Iu. N. BIBIKOV and K.D. SANOVSKII
(Leningrad)
(Received July 2, 1970)

Sufficient conditions are obtained for dissipative behavior of the following equation

$$x^{\cdot\cdot} + f(x^{\cdot}, x) + g(x) = e(t) \tag{1}$$

This equation is called dissipative [1] if for any of its solutions the functions $x(t)$ and $x^{\cdot}(t)$ are uniformly finally bounded for $t \to \infty$. The conditions found here differ from those already known [1, 2] because in the conditions here the functions f and g can be bounded and arbitrarily small in comparison to the force term $e(t)$. Namely, the following theorem is valid.

Theorem. Let the following conditions be satisfied.

1) Piecewise continuous functions $f(z, x)$, $g(x)$ and $e(t)$ are defined for all values of $x, z \in (-\infty, \infty)$ and $t \in [0, \infty)$. These functions ensure the existence of a solution of equation (1) in any point of the phase plane $xx^{\cdot}$ for any $t \geqslant 0$.

2) $e(t) = e_1(t) + e_2(t), \qquad |e_1(t)| \leqslant e_{10} < \infty$

$$E_2(t) = \int_0^t e_2(t)\,dt, \qquad |E_2(t)| \leqslant E_{20} < \infty$$

3) Nondecreasing piecewise continuous functions φ and ψ exist such that

$$\varphi(z) \leqslant f(z, x) \leqslant \psi(z), \qquad x, z \in (-\infty, \infty), \; z\varphi(z) \geqslant 0, \quad z\psi(z) \geqslant 0$$

$$e_{10} < \sup \varphi(z), \quad -e_{10} > \inf \psi(z)$$

4) $xg(x) \geqslant 0$

$$\varliminf_{x\to\infty} g(x) - e_{10} > -\varphi(-2E_{20} - 0) \geqslant 0, \quad \varlimsup_{x\to-\infty} g(x) + e_{10} < -\psi(2E_{20} + 0) \leqslant 0$$

5) $|g(x)| \leqslant g_0 < \infty$

Then equation (1) is dissipative.

Proof 1. Instead of Eq. (1) let us examine the equivalent system

$$\dot{x} = y + E_2(t), \qquad y^{\cdot} = -f(y + E_2(t), x) - g(x) + e_1(t) \tag{2}$$

Let

$$\left.\frac{dy}{dx}\right|_{(2)} = \frac{-f(y + E_2, x) - g(x) + e_1(t)}{y + E_2}$$

In the xy plane let us investigate the curves

$$\Gamma(H, \alpha) = \{(x, y) : {}^1\!/\!_2 y^2 + G(x) + \alpha x = H = \text{const}\}, \qquad G(x) = \int_0^x g(x)\,dx$$

$$-\infty < \alpha, H < \infty$$